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Advancements in the Application of Finite Element Analysis to the Optimization of Composite Yacht Structures

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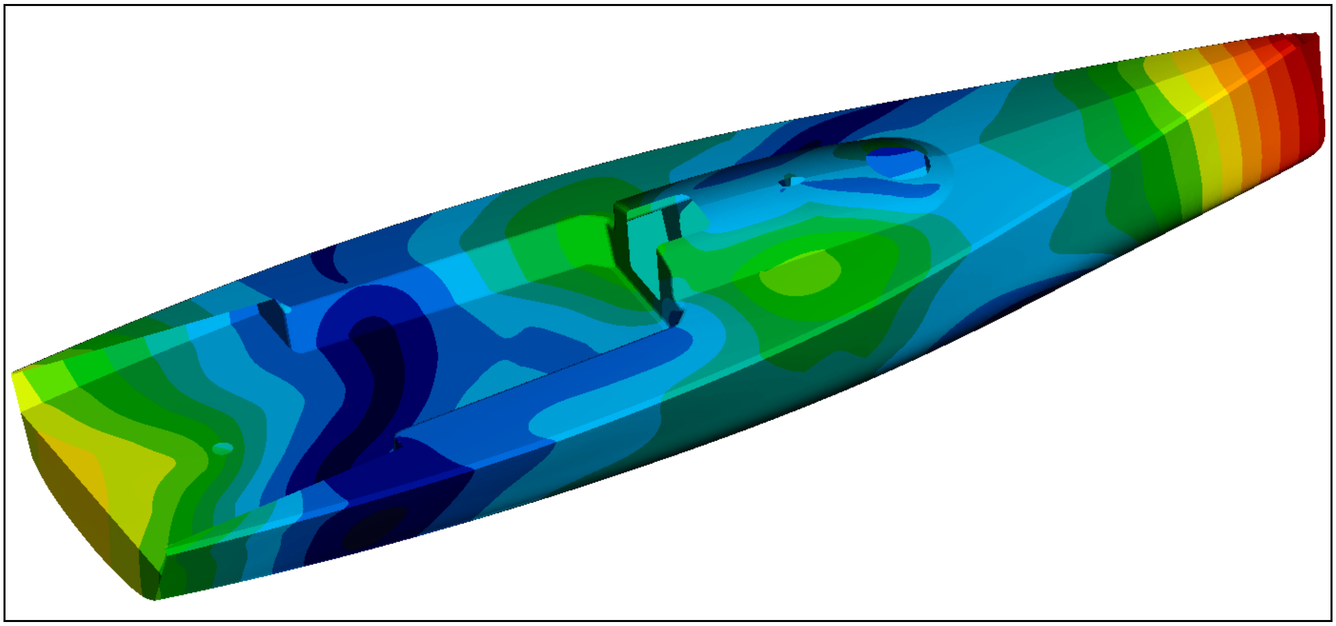

ABSTRACT

Finite Element Analysis (FEA) is mature technology that has been in use for several decades as a tool to optimize structures for a wide variety of applications. Its application to composite structures is not new, however the technology for modeling and analyzing the behavior of composite structures continues to evolve on several fronts. This paper provides a review of the current state-of-the-art with regard to composites FEA, with a particular emphasis on applications to yacht structures. Topics covered are divided into three categories: Pre-processing; Post-processing; and Non-linear Solutions. Pre-processing topics include meshing, ply properties, laminate definitions, element orientations, global ply tracking and load case development. Post-processing topics include principal stresses, failure indices and strength ratios. Non-linear solution topics include progressive ply failure. Examples are included to highlight the application of advanced finite element analysis methodologies to the optimization of composite yacht structures.

INTRODUCTION

Yacht structures provide fertile ground for the application of composite finite element analysis to optimize performance via reduced weight, increased strength and improved durability. The methodologies employed to analyze composite structures have evolved over several decades and continue to be steadily improved and augmented. Advancements on several fronts have enabled more accurate analyses via improvements in meshing; ply and laminate definitions; load cases; methods for failure prediction; and solution types. These improvements have resulted in a continuous advancement in the state-of-the-art in composite yacht structures. This is evident in the many phenomenal record-setting performances that have occurred over the past decade, including breaking the 50 knot speed barrier and the 900+ mile 24-hour run. These records don't come without occasional failures along the way, indicating the need for continued improvement in the ability to accurately design and analyze these lightweight, high performance structures.

PRE-PROCESSING

Pre-processing is the preparation of a model for analysis via definition of geometry, mesh, ply properties, laminate definitions, element orientations and load cases. Although geometry modeling is a crucial first step with many of its own challenges, it will not be discussed in detail here. Advancements in the other key pre-processing areas are discussed below.

Meshing

Advancements in meshing have come about largely as a result of two evolving technologies, namely computing power and auto-meshing quality.

With the speed of contemporary processors, it is effectively not necessary to limit the mesh density of global models. Whereas processor speed and memory limitations in the past often lead to somewhat coarse global models, with local sub-models extracted and refined for detailed analysis, today the necessity for this is greatly reduced if not eliminated altogether. Mesh density for global models can now be fine enough to represent most structural features with sufficient detail to provide an accurate assessment, minimizing the need for further local, detailed modeling. Sizes of typical small structural features (e.g. ring frames, deck beams, unidirectional tape reinforcements, etc…) can often be on the order of 100mm or so. Good modeling practice would suggest three to four elements across the width of any such feature to provide adequate resolution. A global element edge length of 40mm will often provide for good overall model resolution, with refinement down to 20mm or so in these smaller areas where enhanced detail may be required. Figures 1 and 2 show a typical global mesh in the mid-body region of a 42 foot high performance yacht (designed by Mick Price of Weaver Price Design & Construction, Annapolis, MD).

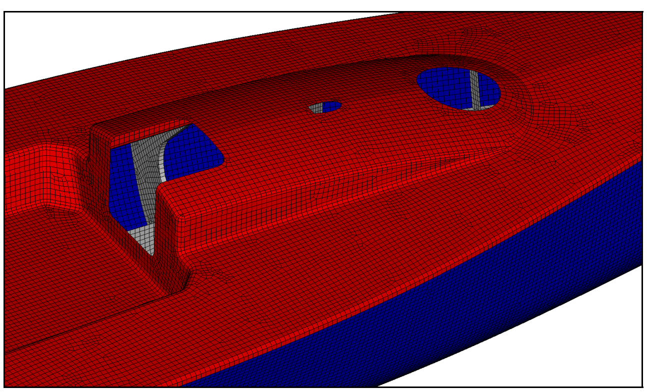

Figure 1 – Typical hull/deck mid-body global mesh

Total shell element count for this model is approximately 85,000. Surface area and hence element count will scale roughly with length to a power of 1.6, yielding for example a typical shell element count of approximately 190,000 for a Volvo 70 and approximately 340,000 for a 100 foot maxi.

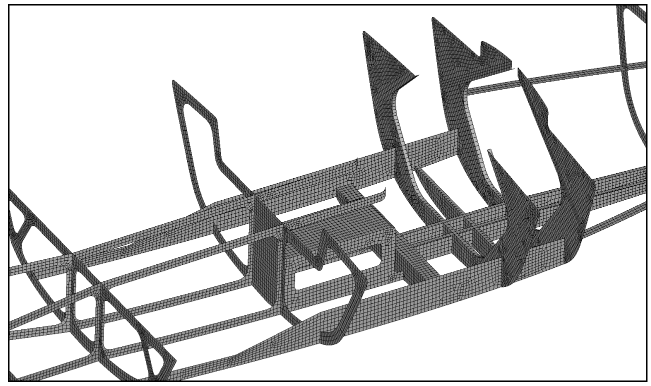

Figure 2 – Typical internal mid-body global mesh

Even for complicated structures with over 150 distinct laminate zones, linear static models of this size will solve within about ten minutes with modern hardware.

The second area of advancement in meshing technology is auto-meshing quality. Auto-meshers are crucial for the efficient and accurate meshing of geometry, removing the tedium of hand-meshing and therefore enabling efficient large-model construction. Beyond the basics of edge length and edge/node connectivity, checks for element quality include assessments of taper, warp, skew, aspect ratio and degeneracy. In some cases, the quality of the underlying surface geometry can affect mesh quality. In problematic cases, surfaces may need to be split and/or re-created to reduce inherent u/v distortion which can affect mesh quality. Areas of high curvature can be particularly challenging and should be carefully inspected for element quality. Figure 3 shows an example of a high-quality, quad-dominant mesh transition in the forward area of the bottom portion of the hull, where a flat bottom aft transitions to a highly curved forefoot and turn-of-bilge up the topsides and stem. Note the absence of any lower-accuracy tri elements and the uniform size and aspect ratio of all quad elements with minimal taper, warp or skew.

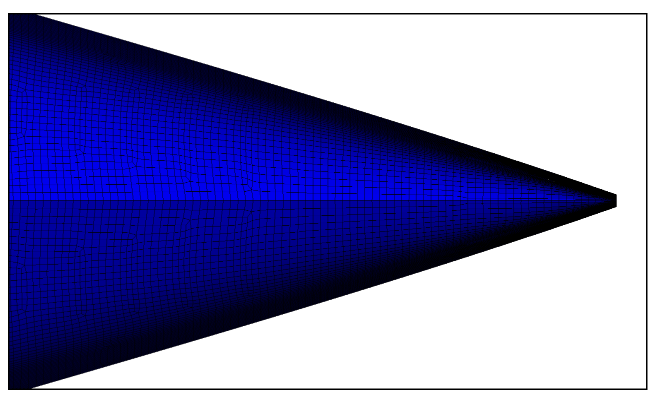

Figure 3 – Quad mesh transition in high curvature area

Ply Properties

Both the advantages and the complexities of composite materials lie in their orthotropic nature – i.e. mechanical properties vary in different directions. This allows for careful tailoring of laminate strength to accommodate changes in the direction of principal loads throughout the structure. Accurate characterization of ply properties is crucial to accurate analysis results and must include correct values for elastic modulii (E_1 and E_2), in-plane shear modulus (G_{12}), through-thickness shear modulii (G_{13}, G_{23}) and in-plane Poisson's ratio (ν_{12}). Ply properties can come from either theoretical calculations or from physical testing. Theoretical calculations of ply properties are based on Micromechanics theory for combining individual matrix and fiber properties to predict cured ply properties. Alternatively, testing of plies can be carried out to characterize strength and stiffness. Testing is especially appropriate for unique combinations of fiber and matrix. Testing is also the best way to characterize the results of a builder's processing techniques - to be sure the engineered product is representative of what the builder can deliver.

The principal advancement in characterizing ply properties over the past decade has been the ever-increasing database of test results available from manufacturers and other sources such as the military (MIL handbook). Wherever possible, test data should be used to generate the ply properties for finite element analysis to ensure the most accurate results. If specific test data are not available, then test data representative of similar plies made using similar processing techniques may be utilized. Generic properties from Micromechanics theory should only be used as a last resort. Figure 4 lists some typical test data for various pre-preg laminates generated by Hexcel, which could be used in the absence of project-specific test data.

HEXCEL — TYPICAL MECHANICAL VALUES ON EPOXY PREPREG LAMINATES

Volume content of fibres: ≈ 60 % (Carbon); ≈ 50 % (E-glass - Aramid)

		UNITS	FIBRES: E-GLASS UD	E-GLASS Fabric	ARAMID UD	ARAMID Fabric	HIGH STRENGTH CARBON UD	HIGH STRENGTH CARBON Fabric	INTERMEDIATE MODULUS CARBON UD	INTERMEDIATE MODULUS CARBON Fabric
Tensile	σ / |||	MPa	1100	600	1100	500	2000	800	2400	900
	σt ≡	MPa	35	550	35	450	80	750	80	850
	E / |||	GPa	43	20	60	30	130	70	170	90
	Et ≡	GPa	8	19	8	30	9	65	9	90
	Poisson's ratio υ		0.28	0.13	0.34	0.2	0.25	0.05	0.27	0.05
Compression	σ / |||	MPa	900	550	250	150	1300	700	1600	800
	σt ≡	MPa	150	500	150	150	250	650	250	750
	E / |||	GPa	42	17	75	31	115	60	150	80
	Et ≡	GPa	10	16	5.5	30	10	55	11	75
Flexure	σ / |||	MPa	1200	700	550	400	1800	1000	1400	1200
	E / |||	GPa	42	20	40	25	120	65	140	75
In-plane shear	σ	MPa	60	55	45	40	95	80	95	80
	G	GPa	4	4.2	2.1	4	4.4	5.5	4.4	5
Interlaminar shear	σ |||	MPa	75	50	60	50	80	70	80	70

Figure 4 – Typical pre-preg ply properties (Hexcel)

Laminate Definitions & Element Orientations

Once plies have been defined with appropriate properties, laminates can then be built. Laminates are combinations of plies (which can include core materials) that represent the total through-thickness construction at each location throughout the structure. Beyond just combining plies in a stacking sequence, the definition of a laminate also importantly includes a material orientation vector and a normal direction vector for every element in the model. Material orientation vectors are in-plane vectors for each element that define the direction relative to which the orthotropic ply properties for the laminate assigned to the element are applied. It is imperative that the material orientation vectors and the laminate stack definition are developed in conjunction such that the resulting properties are correct. An example of the definition of material orientation vectors is shown in Figure 5.

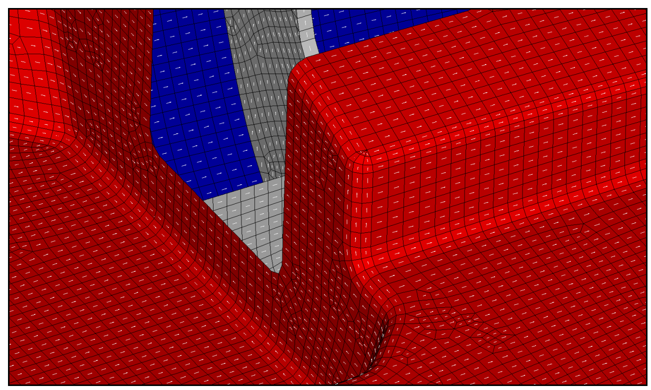

Figure 5 – Element material orientation vectors

Normal direction vectors are out-of-plane vectors for each element that define the direction of the ply stack, typically relative to either a mold surface (hull and deck) or a center-plane (internal structure). An example of the definition of normal direction vectors is shown in Figure 6. Again it is imperative that normal direction vectors are carefully set to represent the correct thickness offset and overall stiffness.

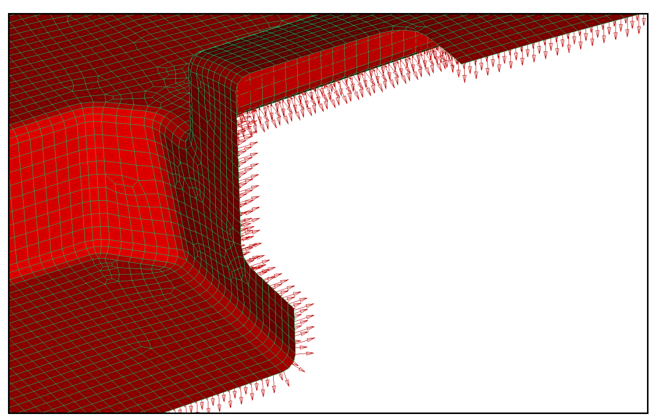

Figure 6 – Element normal direction vectors

Setting normal direction vectors is a fairly straightforward process of defining a back face for each element and/or the surface to which it is attached. In conjunction with an auto-mesher, it is usually most convenient to set normal directions for the surfaces first, which are then inherited by the elements.

Setting material orientation vectors is a more involved process, with different options depending on the complexity of the underlying geometry. The most basic method is vector projection, which may be relative to global coordinate axes or any other vector direction. This is all that is required for flat surfaces and is also usually sufficient for gently curved surfaces (e.g. foredeck, sidedeck, hull bottom aft). Choosing the projection vector for the material orientation must take into consideration the methodology which will be employed to lay plies onto the mold surface. For instance, it can be seen in Figure 5 that the orientation vectors for the deck and cockpit sole are projections of the global x-axis (fore/aft) whereas the orientation vectors for the cabin back are projections of the global y-axis (port/stbd). The cabin-back vectors could also have been projected from the global z-axis (up/down). Either orientation is sufficient as long as the definition of the laminate stack for this region properly accounts for the material orientation vectors. Typically the projection vector would be chosen to correspond to the direction the roll of material is laid onto the mold.

For more highly curved surfaces, vector projection does not adequately account for the way a roll of material will drape over the surface, nor the possibility for in-plane shearing of the fibers to conform to the surface. Both of these characteristics will vary depending on the ply type (e.g. uni-directional, bi-axial, double-bias). To some extent a good builder can mitigate these issues with careful trimming and re-orienting of plies to attempt to maintain conformance to a particular vector projection. In some cases this may be desirable, e.g. for unidirectional reinforcements that are intended to have a specific orientation regardless of the underlying surface curvature. However, this can be a tedious process and in many cases is not required.

Developments over the last decade or so have resulted in more widely available software add-ons to enhance typical pre-processor functionality in order to accurately account for draping over highly curved surfaces. These allow for improved accuracy in determining material orientation vectors that are representative of the way in which the material will actually conform to the mold surface. Note that this can result in a nearly infinite variation in material orientations throughout the model, though most software add-ons allow the specification of user-defined tolerances within which orientations may be considered to be the same (e.g. 3 degrees or so).

Figures 7 and 8 show a comparison of material orientation vectors in the hull forefoot/topsides/stem area achieved using projection (Figure 7) and draping (Figure 8) for a bi-axial ply. Note that the results of the draping process are dependent on the selection of both a starting point and initial direction for laying down each ply. In the example shown, the starting point was at the intersection of DWL and STN 5 and the initial direction was parallel to the x-axis (fwd). This would be fairly typical of how a ply might be laid down on the mold in actual practice. The material shearing angle using the draping method is 11 degrees, as compared to a zero degree (horizontal) projection. For a biaxial carbon ply, this difference in material orientation can result in up to a 34% reduction in ply stiffness and a 44% reduction in ply strength relative to horizontal.

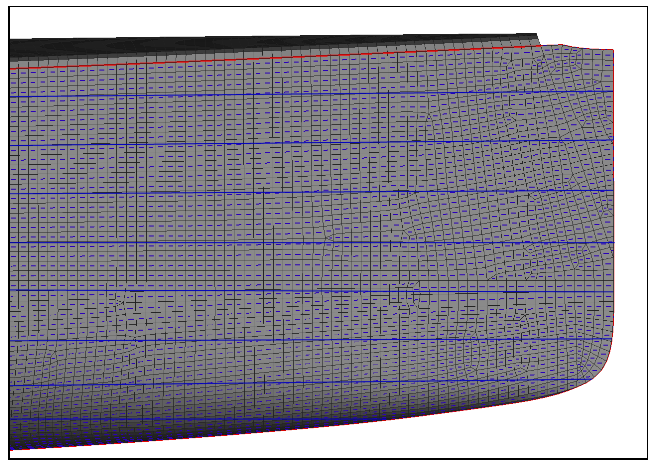

Figure 7 – Material orientation vectors via projection

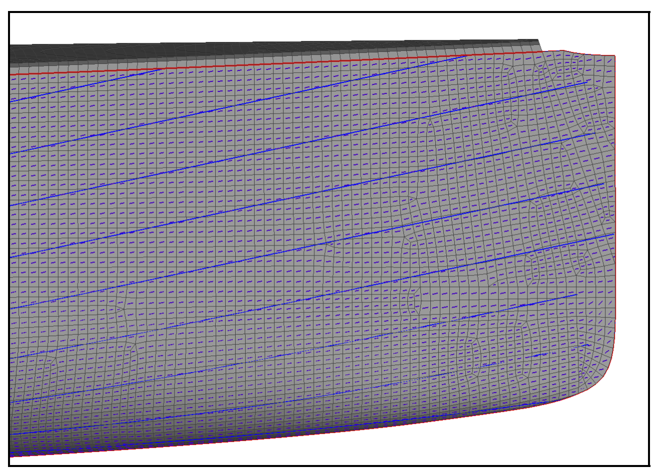

Figure 8 – Material orientation vectors via draping

Draping software add-ons also have the ability to export flat patterns for cutting material from a roll to best conform to a molded shape, or to determine the location for darts (slits) to be added to a standard roll-width to provide optimal conformance to the mold shape.

A further benefit of many add-ons for pre-processors is advanced ply management, which simplifies the definition of layup sequences and the generation of element properties in areas of overlapping reinforcements. Figures 9-13 show a simple example of a flat bulkhead that has a base biaxial / double bias laminate covering the entire bulkhead surface, with local double bias and unidirectional reinforcements added in selected areas.

Figure 9 shows the material orientation vectors, which in this case have been defined as vertical for all elements on the bulkhead. Figure 10 shows the six different laminate property zones created by the application of the various plies. The hatched zones represent areas of overlapping reinforcements where unique properties are created that have stiffness and strength values distinct from the individual plies that are laid down. Figures 11-13 show the sequence of lamination, starting with the base laminate and followed by double bias and unidirectional reinforcements. Note that each unidirectional ply has a direction associated with its application to define the orientation of the fibers relative to the element material orientations. These are then automatically combined in the ply overlap areas and converted to the correct properties for each zone relative the material orientation vectors. In this way, properties for all six distinct laminate zones are created using as input only properties for three basic plies.

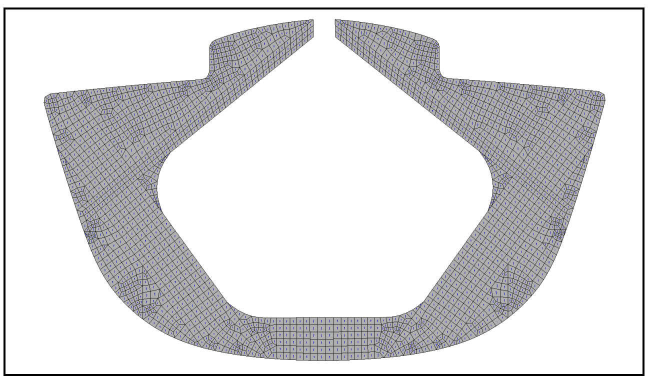

Figure 9 – Material orientation vectors via projection

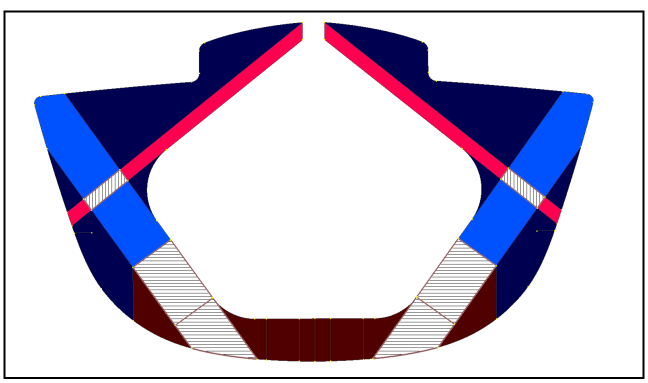

Figure 10 – Distinct element property zones

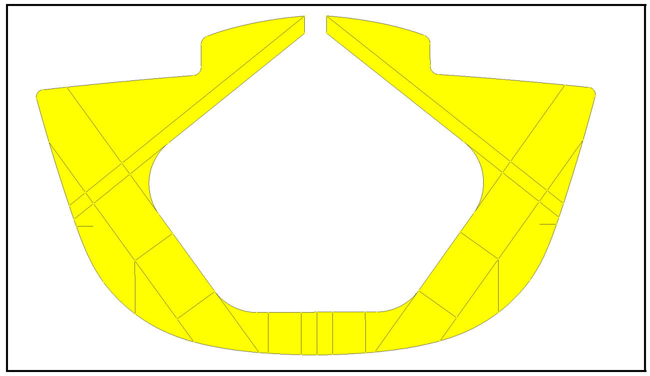

Figure 11 – Base biaxial / double bias laminate

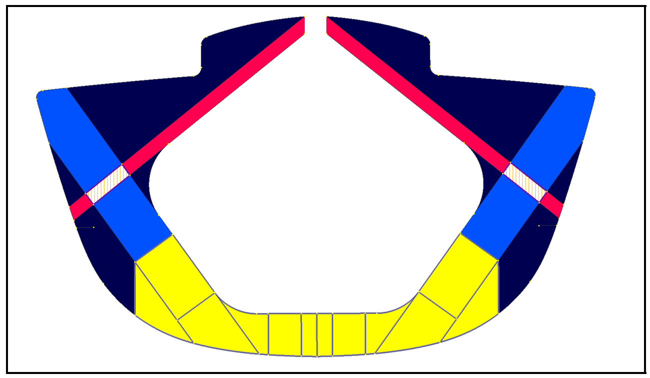

Figure 12 – Double bias laminate reinforcement extents

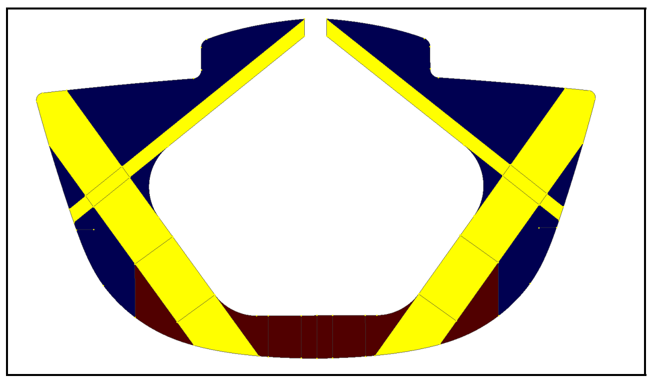

Figure 13 – Unidirectional laminate reinforcement extents

These techniques can significantly reduce the time required to define properties for a complete model, remove the tedium of manually calculating properties in areas of overlapping reinforcements and eliminate potential sources of error in data entry and transfer. Finally, advanced ply management techniques also include global ply tracking, in order to post-process results for a particular ply across multiple laminate zones regardless of where in the local laminate stack the ply may fall (which can vary with the application of local reinforcements).

Load Cases

A complete finite element analysis will consider several load cases to ensure that the structure is adequately designed and optimized for the widest possible range of conditions. A typical suite of load cases might include:

- Dockside rig loads, light rig tension
- Dockside rig loads, max rig tension
- Steady-state upwind
- Steady state reaching/downwind
- Wave slamming/pitching
- 90 degree knockdown
- Grounding

To fully characterize each load case it is necessary to calculate and apply all of the individual loads from the rig, sails, keel, rudder(s), hydrostatic/hydrodynamic forces and crew/gear weights. Load cases should be as completely developed as possible to ensure accurate results free from artificial constraints. Load cases can and should be as fully force and moment balanced as possible, similar to a VPP force and moment balance but in all six degrees of freedom. This requires diligence to collate information from many sources including LPP/VPP, rig analysis, CFD results for hull pressure distributions (if available) and first principles calculations.

	Forces		
	Fx ***(N)***	***Fy*** ***(N)***	***Fz*** ***(N)***
Windward V1 (Stbd)	0	0	-17800
Windward D1 (Stbd)	0	-3380	-11080
Leeward V1 (Port)	0	0	-2200
Leeward D1 (Port)	0	830	-2680
Forestay	-10960	0	-30110
Backstay	4720	0	-12970
Main Traveller Car	0	0	-7920
Main Winch (Stbd)	0	-6905	720
Main Winch (Port)	0	6905	720
Jib Tack	-1535	-195	-3040
Jib Turning Block 1 (Port)	-2185	0	-4070
Jib Turning Block 2 (Port)	-25	1500	-3220
Jib Winch (Port)	4750	-670	0
Mast Base	-1200	0	**82750**
Mast Collar	4925	580	0
Rudder Upper Bearing	0	-785	285
Rudder Lower Bearing	0	1450	-530
Keel Weight	0	**-8037**	**22083**
Hydrostatic Pressure	0	9790	-26780
Self-Weight	0	-7011	14659
Total Load (N)	**-1510**	**-5929**	**-1183**
Balance Accel - Model Mass (mm/s2):	2984	11717	2338
Balance Accel - Actual Mass (mm/s2):	*337*	*1322*	*264*
Balance Accel - Actual Mass (g):	**0.03**	**0.13**	**0.03**

Figure 14 – Upwind load case force balance

When applied forces have been calculated thoroughly, any residual forces and moments can be readily handled using inertia relief to apply small balancing accelerations. Inertia relief is an automated process which calculates and applies linear and rotational accelerations in all six degrees of freedom in order to balance the model and allow its solution without any rigid constraints. It was developed and is widely used for flight structures as well as floating vessels. However, inertia relief should not be used as a crutch to allow inherently unbalanced models to be run.

Spreadsheet-based analysis can be used to estimate force and moment balance prior to load application and can be used as a check on model force and moment summations to validate the modeling process. Figure 14 shows the results of a force balance for a steady-state upwind load case. Moment balances are also similarly calculated with the spreadsheet. Note that the residual balancing accelerations required, if applied to actual boat mass, are on the order of one-tenth gravitational acceleration. Depending on modeling methodology, higher applied accelerations may be required if the model mass is less than the actual boat mass (e.g. since rig and keel loads are typically calculated and applied as forces rather than being represented as mass elements).

In general it is preferable to start with a complete global model with a fully developed, force and moment balanced load case. Local areas of interest can be studied in more detail using sub-modeling techniques, which can be used to extract portions of a global model with force or displacement boundary conditions from the global solution. In some cases it may be desirable to proceed directly with a model of only a portion of the global structure. This might be the case if a global analysis is not required but there is a particular area of interest that merits study. In such instances it will be necessary to apply artificial constraints to the model since it will not be a fully floating structure. If this is done, the model should include enough of the structure surrounding the area of interest so that constraints can be applied without affecting the results in the area of interest. Typically this can be accomplished by placing constraints at least two frames or bulkheads away.

POST-PROCESSING

Post-processing involves the interrogation of the model solution to assess the structural deflection and strength. This process is significantly more complex for composite materials than for isotropic materials due not only to the orthotropic nature of each ply, but also to the many combinations of plies and their interactions with one another across the model. Except in simplistic cases (e.g. single plies or entirely unidirectional laminates) evaluation of the strength of laminates must be carried out using one or more failure theories specific to orthotropic materials.

Standard Composite Failure Theories

Decades of research have yielded a multitude of composite failure theories. Some are general in nature and apply to any orthotropic material, while others have specific limitations (e.g. unidirectional only or specific material types). Figure 15 lists several typical failure theories available in many commercial FEA programs and their mathematical definitions.

Theory	Failure Index	Remarks
Hill	$\frac{\sigma_1^2}{x^2} - \frac{\sigma_1\sigma_2}{x^2} + \frac{\sigma_2^2}{y^2} + \frac{\tau_{12}^2}{s^2} = \text{F.I.}$	Orthotropic materials with equal strengths in tension and compression.
Hoffman	$\left(\frac{1}{x_t} - \frac{1}{x_c}\right)\sigma_1 + \left(\frac{1}{y_t} - \frac{1}{y_c}\right)\sigma_2 + \frac{\sigma_1^2}{x_t x_c} + \frac{\sigma_2^2}{y_t y_c} + \frac{\tau_{12}^2}{s^2} - \frac{\sigma_1\sigma_2}{x_t x_c} = \text{F.I.}$	Orthotropic materials under a general state of plane stress with unequal tensile and compressive strengths.
Tsai-Wu	$\left(\frac{1}{x_t} - \frac{1}{x_c}\right)\sigma_1 + \left(\frac{1}{y_t} - \frac{1}{y_c}\right)\sigma_2 + \frac{\sigma_1^2}{x_t x_c} + \frac{\sigma_2^2}{y_t y_c} + \frac{\tau_{12}^2}{s^2} + 2F_{12}\sigma_1\sigma_2 = \text{F.I.}$	Orthotropic materials under a general state of plane stress with unequal tensile and compressive strengths.
Max Stress	$\text{Max}\left[\left(\frac{\sigma_1}{X_t}\right), \left(\frac{\sigma_2}{Y_t}\right), \left(\frac{\lvert\tau_{12}\rvert}{S}\right)\right]$	None
Max Strain	$\text{Max}\left[\left(\frac{\varepsilon_1}{X_t}\right), \left(\frac{\varepsilon_2}{Y_t}\right), \left(\frac{\lvert\gamma_{12}\rvert}{S}\right)\right]$	None

Figure 15 – Composite failure theories (NEi/NASTRAN)

The Maximum Strain and Maximum Stress theories are termed *non-interactive* since they evaluate the effects of the two orthogonal in-plane principal strains/stresses and the in-plane shear strain/stress in isolation from one another, with failure predicted based on any one of the three strain/stress levels exceeding the ply limit for that quantity. While these theories do not accurately predict failure for multi-axial stress states, they can still be useful as a means for evaluating principal load directions and vectors associated with these constituent strains/stresses can be used as a guide for the application of reinforcements to best handle areas dominated by highly directional loads.

The Hill, Hoffman and Tsai-Wu theories are all *interactive*, in that they consider the combined effects of the principal and shear strains, with failure predicted based on some combination of their effects. Note that in each case the failure index (FI) is a scalar quantity, similar to von Mises stress for orthotropic materials. A failure index of unity indicates that the material has begun to fail (usually first ply failure). A failure index less than one indicates that the combined stress state is less than that required for failure. A failure index in excess of one indicates failure has occurred. Similar to material non-linearity beyond yield strength for an orthotropic material, the solution for a composite material is essentially non-linear once the failure index has exceeded unity. A subsequent section will elaborate upon one option for non-linear analysis of laminates when failure indices exceed unity.

Note that it is sometimes more convenient to report the results of failure analysis as a stress ratio (SR), which is essentially the reciprocal of failure index and acts as a calculation of safety factor. Note also that each ply within all the laminate zones throughout the model must be evaluated against failure criteria. Typically, initial plots of maximum (of all plies) failure index will indicate areas of concern, which can then be more closely interrogated on a ply-by-ply basis to determine which plies are failing (or have sub-standard margins) and what corrective action can be taken.

Advanced Composite Failure Theories

Several new composite failure theories have emerged over the past decade that promise ever-increasing accuracy in the assessment of laminate strength. Two theories of note will be discussed here: LaRC02 and Multi-Continuum Theory (MCT).

LaRC02 Failure Theory

The LaRC02 composite failure theory was developed by researchers at NASA's Langley Research Center. It attempts to combine the best contributions from several other theories and methodologies to arrive at discrete predictions for failure of both fiber and matrix constituents in both tension and compression. The theory leverages contributions from the Hashin, Puck, Mohr-Coulomb and Maximum Strain theories. The LaRC02 theory is restricted to unidirectional plies. Reference 1 outlines the theory.

The following series of images illustrates a comparison of the Tsai-Wu theory versus the LaRC02 theory as applied to an area of unidirectional reinforcement. Figures 16 and 17 highlight regions of the deck where unidirectional reinforcements have been applied in the longitudinal and transverse directions, respectively. The longitudinal reinforcements contribute to global bending stiffness which is critical to maintaining adequate forestay tension which in turn is a major determinant in maintaining accurate sail shape. The transverse reinforcements stiffen the mid-body section against the combined effects of shroud tension (acting upward) and mast compression (acting downward). These coupled loads can induce large deflections through the mid body area as the shrouds attempt to pull their supporting frame and the hull topsides upward at the sheer while the mast attempts to push the mast step and its supporting internal structure downward.

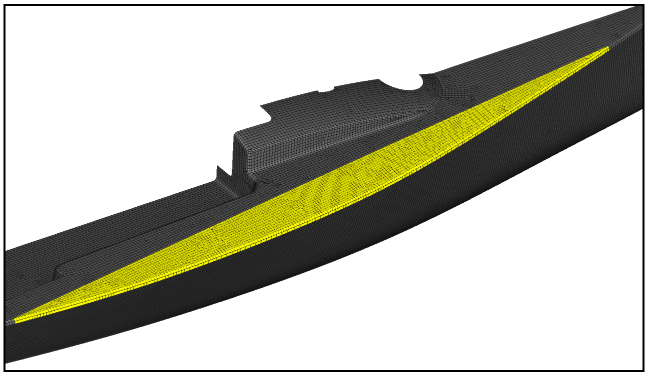

Figure 16 – Deck longitudinal unidirectional reinforcement

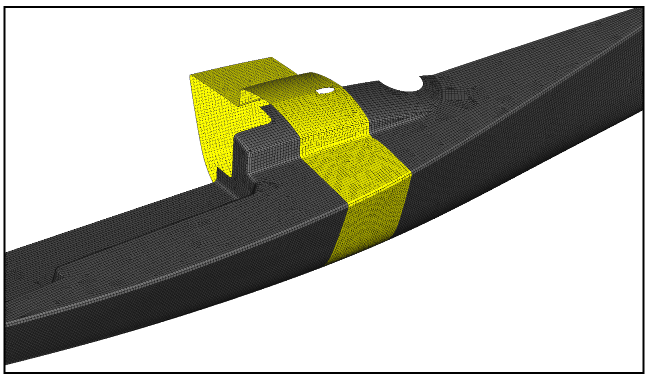

Figure 17 – Deck transverse unidirectional reinforcement

In addition to these unidirectional reinforcements, there is a base laminate composed of inner and outer plies of both biaxial and double-bias material that covers the entire extents of the deck either side of a core material. The resulting stress state in the area of overlap in way of the shrouds and mast is quite complex and not easily ascertained using only a principal stress analysis.

Figure 18 shows a plot of the Tsai-Wu failure index for the base laminate, outer skin biaxial ply. Note that for this and subsequent images in this section, failure indices are shown with a contour scale ranging from 0 (blue) to 0.25 (red).

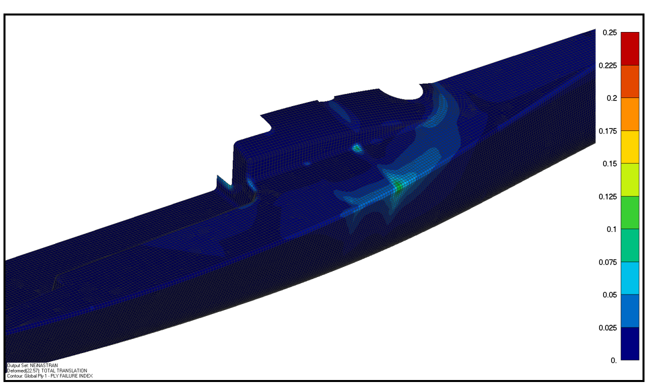

Figure 18 – Base laminate biaxial ply, Tsai-Wu FI

It can be seen that this biaxial ply is stressed well within its safe range, with a maximum failure index of approximately 0.1. There are minor concentrations evident near the forward and aft extents of the transverse unidirectional reinforcement, which are just forward and aft of the internal frames supporting the mast and chainplates.

Figures 19 and 20 show plots of failure index for the longitudinal unidirectional, outer skin reinforcement ply for the Tsai-Wu and LaRC02 failure criteria, respectively. The Tsai-Wu criterion predicts a failure index of approximately 0.1 (similar to that predicted for the biaxial ply) whereas the LaRC02 criterion predicts a failure index approximately double this, around 0.2. Additionally, the LaRC02 plot shows significantly more area of the reinforcement experiencing higher than trivial stresses. It also displays clear evidence of the boundary effects due to the crossing transverse unidirectional reinforcement and the internal frames. Note that the LaRC02 plot is showing specifically fiber compression failure index. A comparison to plots for fiber tension, matrix compression and matrix tension reveals that fiber compression is the dominant failure mode. This makes intuitive sense as we know that global bending due to rig and keel loads will put the side deck in compression. One salient feature of the LaRC02 criteria is that it takes into account fiber buckling when assessing fiber compression failure.

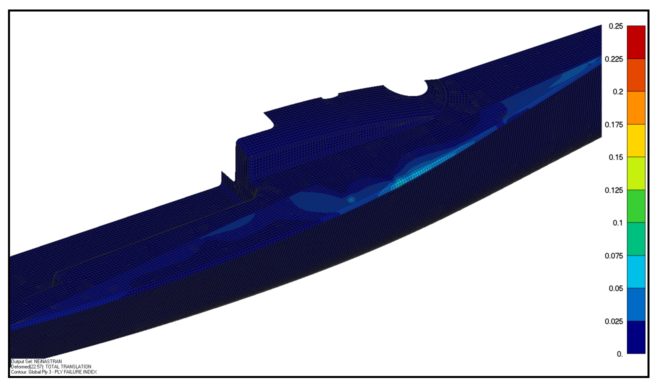

Figure 19 – Unidirectional ply, Tsai-Wu FI

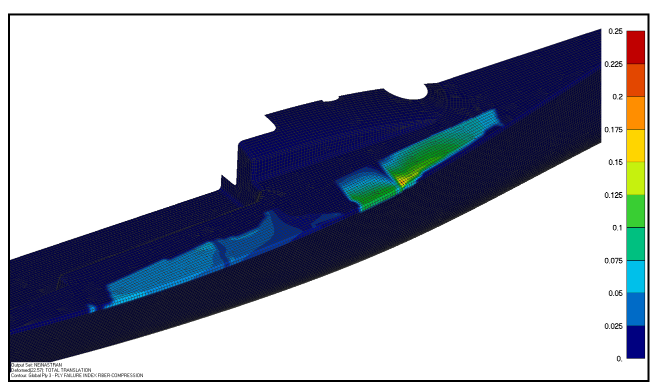

Figure 20 – Unidirectional ply, LaRC02 FI (Fiber Comp.)

Although the Tsai-Wu criterion does consider both compressive and tensile stresses in the calculation of ply failure, it makes no distinction between compressive and tensile failure modes. Furthermore, the Tsai-Wu criterion makes no distinction between fiber and matrix failure. It treats the ply as having "smeared" properties that are the result of the constituent fiber and matrix properties. In fact, it is really a generic failure criterion for any orthotropic material, whether composed of polymer reinforced fibers or not. This makes it broadly applicable and hence it has gained widespread acceptance for use in composites analysis. However, as this example illustrates, it is not as accurate in some situations.

The following series of images shows a similar comparison for the area of the hull in way of the keel and mast base that is reinforced with both longitudinal and transverse unidirectional plies. Figure 21 shows the extents of the reinforcement plies (note that the internal structure on the starboard side has been hidden from view in order to better illustrate the hull laminates in the area of interest). Figure 22 shows a plot of the Tsai-Wu failure index for the base laminate, outer skin biaxial ply. Figure 23 shows a plot of the Tsai-Wu failure index for the outer skin, longitudinal unidirectional reinforcement ply. As noted above, the Tsai-Wu index is a single assessment of the combined fiber and matrix stresses in tension, compression and shear.

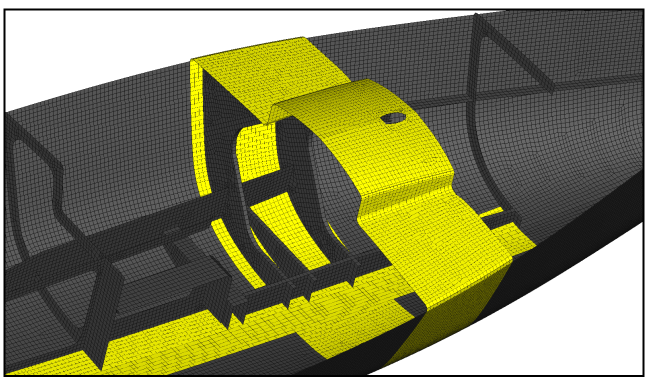

Figure 21 – Hull trans/long unidirectional reinforcements

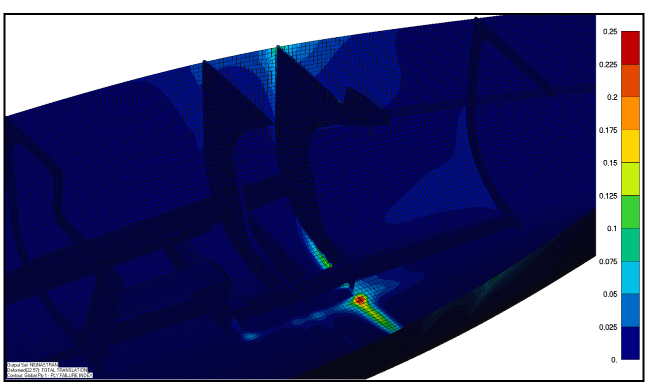

Figure 22 – Base laminate biaxial ply, Tsai-Wu FI

Figures 24 and 25 show the LaRC02 failure indices for fiber and matrix tension in the unidirectional ply. Again this yields more insight into the behavior of the constituent components of the laminate. In particular, it should be noted that the failure mode is predicted as matrix-dominated, rather than fiber-dominated. The Tsai-Wu criterion does perform reasonably well in this case, as the contour shown in Figure 23 can be seen to reflect a combination of the two LaRC02 failure modes shown in Figures 24 and 25. However, no insight is gained into the individual fiber and matrix behavior as with LaRC02.

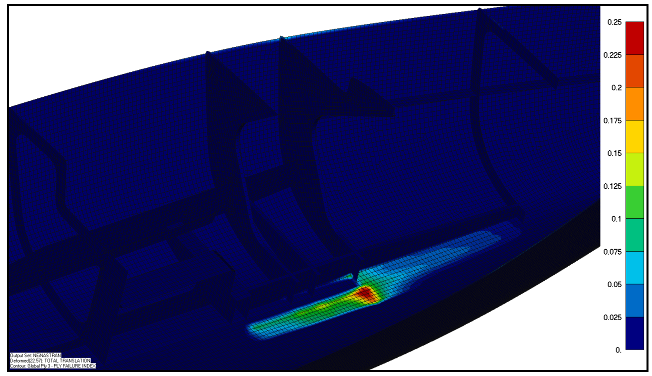

Figure 23 – Unidirectional ply, Tsai-Wu FI

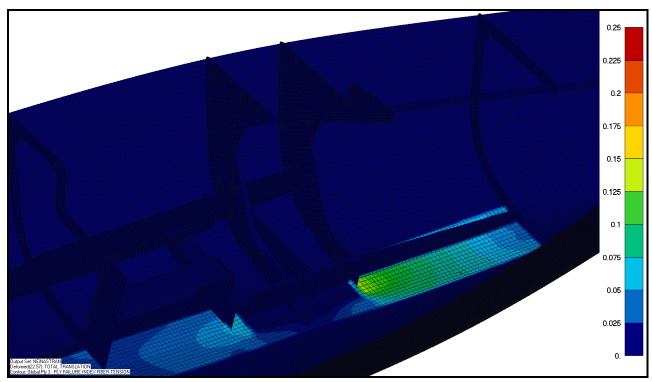

Figure 24 – Unidirectional ply, LaRC02 FI (Fiber Tens.)

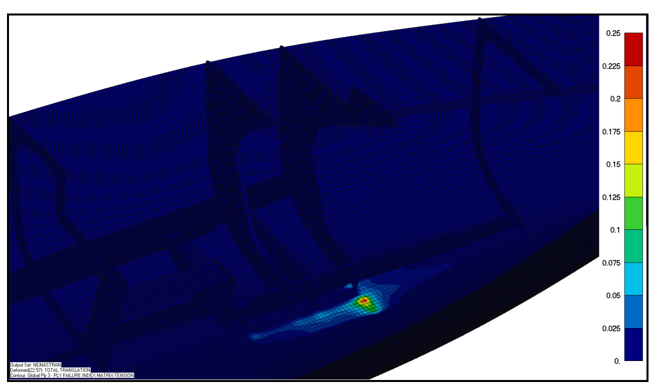

Figure 25 – Unidirectional ply, LaRC02 FI (Matrix Tens.)

Multi-Continuum Failure Theory (MCT)

Another failure theory gaining increasing acceptance is the Multi-Continuum Theory developed by Firehole Technologies and researchers at the University of Wyoming. MCT is similar to LaRC02 in that it decomposes the ply strains into constituent fiber and matrix stresses. One advantage of MCT over LaRC02 is that it is applicable to both woven and biaxial plies as well as to unidirectional plies. This gives it potentially wider general applicability. However, MCT does require some additional material characterization data. Although not cumbersome, this does add a small amount of additional complexity to the modeling process. The principles of the theory are outlined in References 2 and 3. Additional details of its applicability to progressive ply failure analysis and fatigue analysis are discussed in References 4 and 5.

Figures 26-27 show plots of the MCT fiber failure index in the deck reinforcement zone. Note that in all of the plots for the MCT failure indices the contour scale ranges from 0 (blue) to 0.0625 (red). This is due to the fact that the MCT output is quadratic in nature – i.e. SR = 1/SQRT(FI) and SQRT(0.0625) = 0.25. Thus this scale provides the correct comparison to the previous images.

Figure 26 shows the biaxial ply of the base laminate (FI for the fibers in the element material -1 longitudinal direction). This plot can be compared to Figure 18 for the Tsai-Wu theory. While the overall nature of the failure index contour is similar, the MCT theory predicts a notably higher relative overall level of failure index. Figure 27 shows the longitudinal unidirectional reinforcement ply (FI for the fibers in the element material -1 longitudinal direction). This plot can be compared to Figure 19 for the Tsai-Wu theory. The trend is similar to that for the biaxial ply, with the MCT theory predicting a similar failure index contour but a higher overall level of failure index. In this case, however, the difference in magnitude is not as notable as that for the biaxial ply.

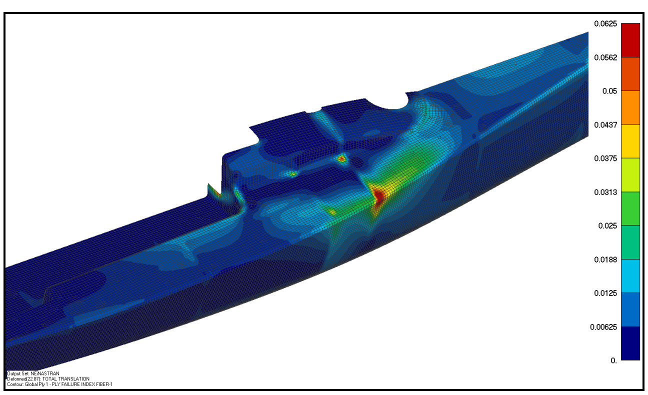

Figure 26 – Biaxial ply, MCT FI (Fiber 1)

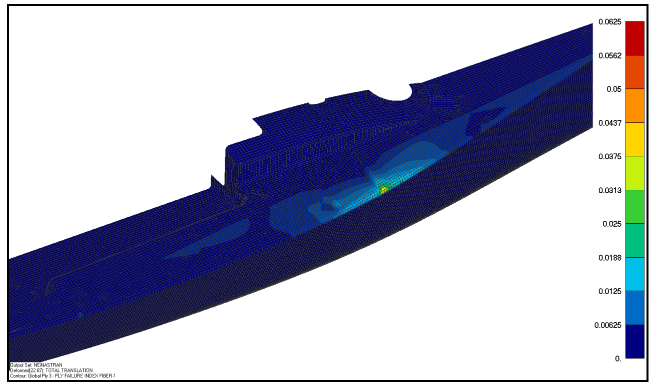

Figure 27 – Unidirectional ply, MCT FI (Fiber 1)

Figures 28-29 show plots of the MCT fiber and matrix failure index in the hull longitudinal unidirectional reinforcement ply. These images are comparable to those shown for the LaRC02 failure theory in Figures 24-25.

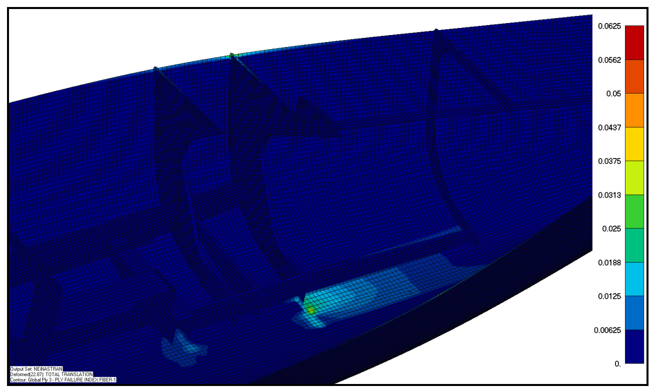

Figure 28 – Unidirectional ply, MCT FI (Fiber 1)

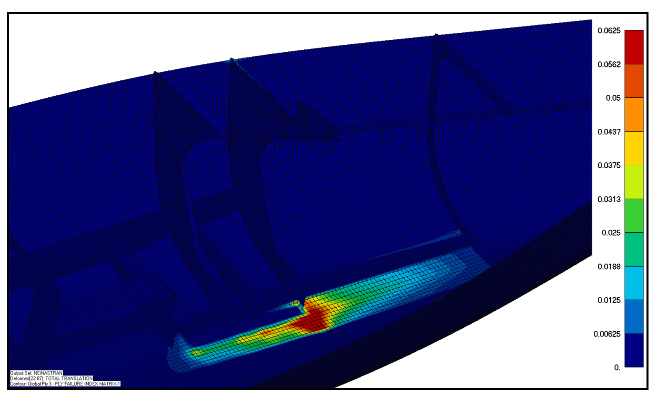

Figure 29 – Unidirectional ply, MCT FI (Matrix 1)

Again the overall trends are similar, but it is interesting to note that MCT predicts relatively lower failure indices for the fiber and relatively higher indices for the matrix. It is also noteworthy that the results from Tsai-Wu shown in Figure 23 seem to compare favorably to the MCT results.

Failure Theory Selection

The comparison of failure index results in the preceding sections begs the obvious question - which one is right? Unfortunately, there is no simple answer to this question. Validation of composites failure theories is an area of intensive ongoing research and a complete discussion is well beyond the scope of this review. There have been several published efforts to make meaningful standardized comparisons between various failure theories and test results. The most notable of these is the so-called "World Wide Failure Exercise" (WWFE) by Soden, Hinton and Kaddour, which is presented in References 6 and 7. An additional assessment can be found in Reference 8 by Icardi, Locatto and Longo.

A typical comparison of failure theory predictions to test results is shown in Figure 30, taken from Reference 9.

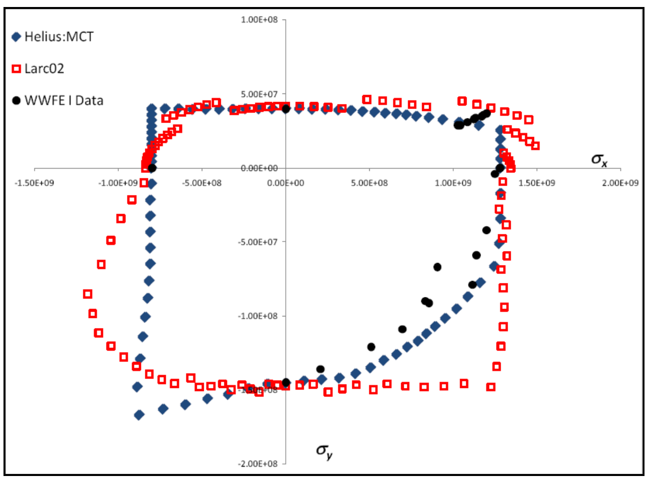

Figure 30 – σx:σy Failure of 0° E-Glass/Epoxy Lamina – MCT vs. LaRC02 vs. Test Data (Firehole Technologies)

Similar results are available for a wide range of different laminates and one could select a different failure theory for different analyses or even different portions of the same analysis based on which theory has better correlation to test data for the given type of laminates and loading being studied. This could be a reasonable approach for small models with few laminates using standard ply configurations and simple loadings. However, as a general rule, a complete model of a yacht structure with many tens or hundreds of laminates and several load cases is sufficiently complex that following such an approach will quickly become impractical.

In the context of the results presented in this paper and for models of similar type and scope, a practical approach might be to first post-process the results using Tsai-Wu, which is generally accepted as being reasonably accurate, conservative and widely applicable. The single-number failure index provided by Tsai-Wu makes manageable the review of large models with many laminates and load cases within a reasonable amount of time. Areas of interest based upon this first-pass assessment could then be studied in more depth using an advanced theory such as LaRC02 or MCT in order to investigate more closely the individual constituent fiber and matrix behavior and to refine any subsequent corrective actions to be taken for any areas found to be deficient. Such a combined approach would provide for reasonably quick initial assessment - which may be all that is required for first-pass iterations - followed by more detailed investigation for final-pass runs.

NON-LINEAR SOLUTIONS

Most analyses for composites will involve linear static solutions, where displacements are relatively small and strains are within the elastic range of the materials. Linear static analysis for composite materials leads to assessment of strength on the basis of first-ply failure.

Progressive Ply Failure Analysis

One useful application for a non-linear solution is progressive ply failure analysis (PPFA), which allows for the sequential degradation of stiffness for the first and subsequent plies failing until complete failure of the laminate has occurred. The solution proceeds by applying load in a step-wise fashion. When a ply is detected as having failed in an element according to a pre-set failure criteria, the stiffness of that element is then degraded to a small percentage of its original value. The solution then proceeds with the next load increment. The process continues in this manner at each load step, sequentially degrading the stiffness of any failing plies, until the full load has been applied. The results of PPFA are a better understanding of the nature of failure in a given area and the amount of reserve strength following initial ply failure.

In general, good design practice will call for high margins on first-ply failure for all plies under all typical loading conditions. However, in some instances where unidirectional reinforcements are added for strength or stiffness under certain load conditions that yield principal stresses in the direction of the uni fibers, other less critical or unusual load cases may strain those reinforcements in the perpendicular direction, leading to matrix cracking due to their low strength in the cross-fiber direction. In such instances, it can be valuable to know the post-failure performance of the remainder of the laminate, which may in fact continue to perform acceptably.

PPFA will not be discussed in further detail here, but References 4 and 10 are recommended to readers interested in a more complete description of the process along with some relevant examples.

CONCLUSIONS

Finite element analysis of composite yacht structures is a complex undertaking. Properly characterizing and analyzing composite materials is a significantly more involved process than analysis of isotropic materials. Recent advancements in the methodologies employed for analyzing composites have provided some simplifications to model construction and management, but have also provided more options for failure analysis. The astute practitioner must continually keep abreast of these developments in order to ensure the most accurate possible solutions to these complex problems.

ACKNOWLEDGEMENTS

The analyses presented here were performed using the following software packages:

Pre- and Post- Processor: FEMAP

Draping and Ply Management: SIMULAYT CMF

Solver: NEi/NASTRAN

Advanced Failure Analysis: HELIUS:MCT

NOTATIONS

E_1 = Elastic Modulus in 1-fiber direction
E_2 = Elastic Modulus in 2-fiber direction
G_{12} = In-plane shear modulus
G_{13}, G_{23} = Through-thickness shear modulii
ν_{12} = Poisson's ratio
σ_{1t} = Tensile strength in 1-fiber direction
σ_{1c} = Compressive strength in 1-fiber direction
σ_{2t} = Tensile strength in 2-fiber direction
σ_{2c} = Compressive strength in 2-fiber direction
τ_{12} = In-plane shear strength
FI = Failure Index
SR = Strength Ratio

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